

Avoiding the uncertainty from correlation between $|\Delta m_{31}^2|$ and CP phase δ in $\nu_\mu \rightarrow \nu_\mu$ long baseline experiments

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Abstract

We introduce a new index $I_{\Delta m_{31}^2}$ to find where is the better setup of the baseline length and energy to avoid as well as possible the uncertainty from the correlation between Δm_{31}^2 and $\cos \delta$ in $\nu_\mu \rightarrow \nu_\mu$ long baseline experiments.

Detection of the CP effect in lepton sector (MNS matrix[1]) is one of the remaining most important subjects in not only elementary particle physics but also particle cosmology. To confirm the existence of the CP phase, many long baseline experiments[2, 3, 4, 5] by using $\nu_\mu \rightarrow \nu_e$ oscillation mode are proposing. After finding the CP effects in $\nu_\mu \rightarrow \nu_e$, as the next step, it will be an important subject to check whether they are consistent with the standard model(SM). To do so, we need to measure the CP effects(phase) independently by using the other oscillation mode. Measuring CP phase by $\nu_\mu \rightarrow \nu_\mu$ mode is going to be more important to confirm the SM and to investigate the existing possibility of new physics. We have to confirm the consistency and the unitarity in lepton sector[6] by comparing the observables extracted from the different oscillation modes. We are investigating the CP effect in $\nu_\mu \rightarrow \nu_\mu$ mode in our work[7]. The dependence of the probability on the CP phase δ with the maximal 2-3 mixing $\theta_{23} = 45^\circ$ is written as follows[7, 8, 9]:

$$\begin{aligned} P_{\mu\mu} &= A_{\mu\mu} \cos \delta + C_{\mu\mu} + D_{\mu\mu} \cos 2\delta \\ &\simeq A_{\mu\mu} \cos \delta + C_{\mu\mu} + O(\sin \theta_{13} \Delta m_{21}^2), \end{aligned} \quad (1)$$

where $D_{\mu\mu}$ as a coefficient of $\cos 2\delta$ is negligible because the magnitude should be proportional to the quite small parameter $\sin \theta_{13} \Delta m_{21}^2$. $A_{\mu\mu}$ and $C_{\mu\mu}$ are the quantities determined by the parameters except for the CP phase δ . The effect from $\cos \delta$ depends on the magnitude of $A_{\mu\mu}$ so that it is an index to know the CP dependence. We discussed where is better set of the baseline length L and the neutrino energy E to extract the CP effect from $\nu_\mu \rightarrow \nu_\mu$ experiment

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and pointed out it favors $E < 2\text{GeV}$, $L > 2000\text{km}$. At once, we showed it seems to be difficult to determine the CP phase because there is a correlation between Δm_{31}^2 and $\cos \delta$ in small L/E [7, 10].

In this letter, we introduce a new index $I_{\Delta m_{31}^2}$ to look for the better region in (E, L) plane and to avoid the uncertainty from Δm_{31}^2 - $\cos \delta$ correlation. Here we are using the following input parameters: $\Delta m_{21}^2 = 8.1 \times 10^{-5} \text{eV}^2$, $\sin^2 \theta_{12} = 0.31$, $\sin^2 2\theta_{23} = 1$, and for an unknown parameter θ_{13} , the upper bound[11] $\sin^2 2\theta_{13} = 0.16$ is used. In the estimation of the probability $P_{\mu\mu}$, we are using the exact solution for the neutrino oscillation in matter [8, 12].

The Δm_{31}^2 - $\cos \delta$ correlation is plotted in Fig.1, where we assume $(\cos \delta, \Delta m_{31}^{2 \text{ true}}) = (0, 2.5 \times 10^{-3} \text{eV}^2)$ as the true values and the plotted points show where the probability $P_{\mu\mu}$ at the fake values $(\cos \delta', \Delta m_{31}^2)$ are almost same with $P_{\mu\mu}^{\text{true}}$ at true value. The figure shows the linear relation between the fake parameters Δm_{31}^2 and $\cos \delta'$. As we discussed in our previous work[7], there is a relation between the true value and fake one which are producing same probability approximately within the $L/E \ll 1000$ as follows:

$$(|\Delta m_{31}^2| - |\Delta m_{31}^{2 \text{ true}}|) = -4J_r \Delta m_{21}^2 (\cos \delta' - \cos \delta) \quad (2)$$

$$= -0.0146 \times 10^{-3} (\cos \delta' - \cos \delta), \quad (3)$$

where $J_r = \frac{1}{8} \sin 2\theta_{12} \sin 2\theta_{23} \sin 2\theta_{13} \cos \theta_{13} \simeq 0.045$. If the relation are satisfied, it means one can not determine the magnitude of CP phase without uncertainty. Namely, for the error of $|\Delta m_{31}^2|$, all range of 360° is satisfied as the solution. Indeed, even if the error is 1% level, $|\Delta m_{31}^2| = (2.50 \pm 0.02) \times 10^{-3} \text{eV}^2$, we have to consider the uncertainty. From the left of Fig.1, one can find

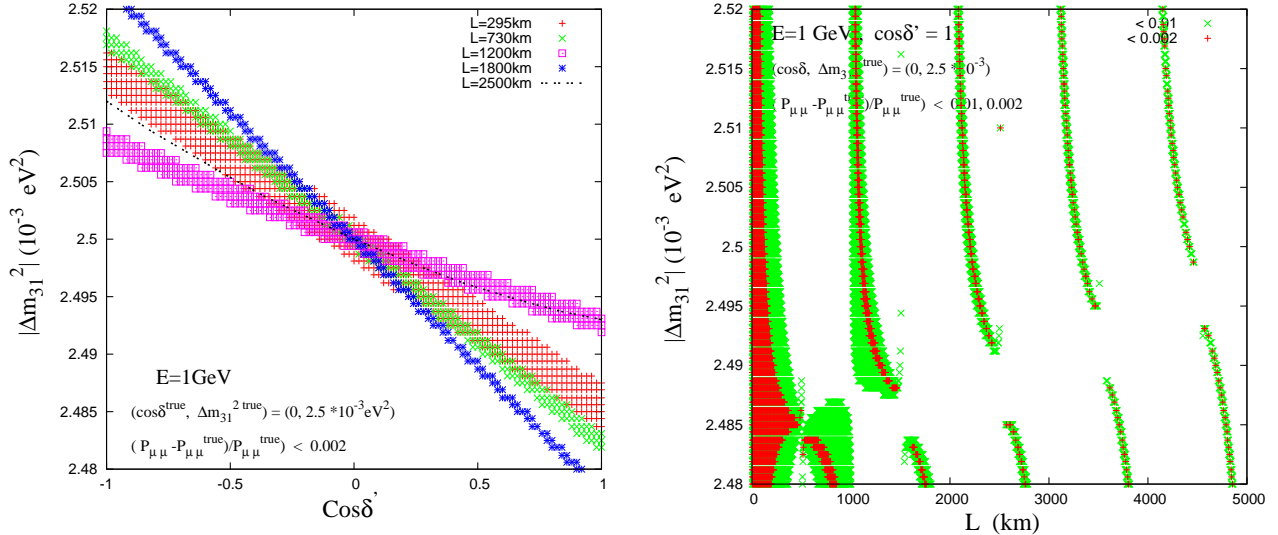


Figure 1: The region shows where $(P_{\mu\mu}(\delta', |\Delta m_{31}^2|) - P_{\mu\mu}^{\text{true}})/P_{\mu\mu}^{\text{true}}$ is smaller than 0.002(Left) and 0.01, 0.002(Right) for $\cos \delta^{\text{true}} = 0$ and $\Delta m_{31}^{2 \text{ true}} = 2.5 \times 10^{-3} \text{eV}^2$ at $E = 1\text{GeV}$. The left is that for $(\cos \delta', \Delta m_{31}^2)$ at several baseline length L and the right is for $(L, \Delta m_{31}^2)$ with $\cos \delta' = 1$.

the linear relation between $|\Delta m_{31}^2|$ and $\cos \delta'$ for several baseline lengths. The right figure shows the dependence of fake region of $|\Delta m_{31}^2|$ on the baseline length L at the case of $\cos \delta' = 1$ ($\delta' = 0^\circ$) which leads to same probability $P_{\mu\mu}$ with true (input) value $\cos \delta = 0$ ($\delta = 90^\circ$). From this, we find that the dependence may not be so trivial. One can find that the fake region breaks at several

L s in the right of Fig.1. Hence we investigate around $L = 505, 1000, 2000, 3000, 4000, 5000(\text{km})$ where the fake region disappears.

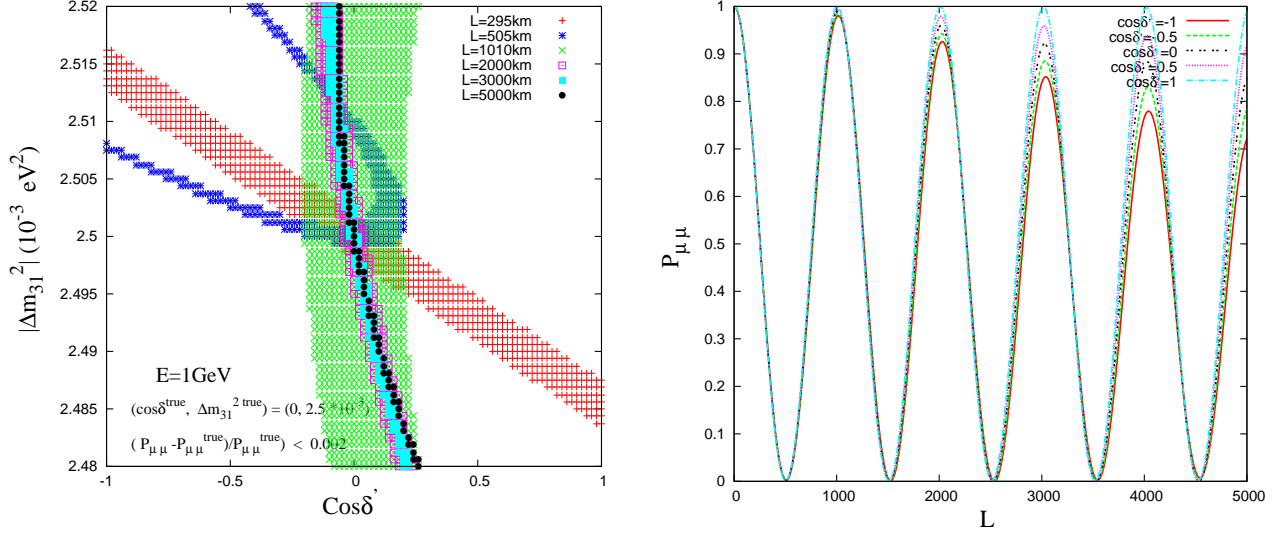


Figure 2: The region satisfying $(P_{\mu\mu}(\delta', |\Delta m_{31}^2|) - P_{\mu\mu}^{\text{true}})/P_{\mu\mu}^{\text{true}} < 0.002$ on $(\cos \delta', \Delta m_{31}^2)$ is shown in the left figure. The region shows almost same probability with it at $\cos \delta^{\text{true}} = 0$ and $\Delta m_{31}^2 = 2.5 \times 10^{-3} \text{eV}^2$ for several L s. The probability as a function of L at $\delta = 0^\circ, 90^\circ, 180^\circ$ in the right.

At $L = 505, 1000, 2000, 3000, 4000, 5000(\text{km})$, the correlation of $|\Delta m_{31}^2|$ and $\cos \delta'$ are plotted in Fig.2(left). The dependence seems to be different with the case in Fig.1 and the almost plotted points are around the true value $\cos \delta = 0$ at the L where the fake regions disappear in Fig.1. It may show that at the several suitable L one can investigate the CP phase without depending on the error of $|\Delta m_{31}^2|$ so strong. Where is the region on (E, L) ? Comparing the right one of Fig.1 with the Fig.2(right) which shows the dependence of the probability on the baseline length L , where the probability shows maximal or minimal. The fake regions also break around $L = 500, 1500, 2500, 3500, 4500(\text{km})$ but the probability is almost 0 so that we can not extract the CP effect around the L . On the other hand, at $L = 1010, 2000, \dots$, $P_{\mu\mu}$ shows maximal and the CP effects will also be maximal so that it may be possible to determine the CP phase without depending on Δm_{31}^2 so hard. They correspond to the region with large $A_{\mu\mu}$. From the left of Fig.2, one can find the extracted solutions of $\cos \delta'$ is around the true value for the error of Δm_{31}^2 at the special L .

There is the uncertainty in determination of CP phase in $\nu_\mu \rightarrow \nu_\mu$ oscillation experiments because of Δm_{31}^2 - $\cos \delta$ correlation. So we introduce a new index $I_{\Delta m_{31}^2}$ to search for where is more suitable energy E and distance L to avoid the uncertainty. It is defined by the difference of maximum and minimum probabilities ($P_{\mu\mu}^{\text{max}}$ and $P_{\mu\mu}^{\text{min}}$) within the error of Δm_{31}^2 ($|\Delta m_{31}^2| = 2.50 \pm 0.02 \times 10^{-3} \text{eV}^2$)¹.

$$I_{\Delta m_{31}^2} = \frac{P_{\mu\mu}^{\text{max}}(|\Delta m_{31}^2|) - P_{\mu\mu}^{\text{min}}(|\Delta m_{31}^2|)}{P_{\mu\mu}^{\text{max}}(|\Delta m_{31}^2|) + P_{\mu\mu}^{\text{min}}(|\Delta m_{31}^2|)}, \quad (4)$$

This is the index to indicate how affecting the probability from the error of $|\Delta m_{31}^2|$. The regions which the new index is as small as possible are favored to avoid the effects from $|\Delta m_{31}^2|$. On the

¹We expect that the experimental error of Δm_{31}^2 will be reduced up to 1% level in the future experiments.

other hands, to determine the CP phase, the regions the dependence on $\cos \delta$ becomes larger are favored. Using $A_{\mu\mu}$ one can find the regions. $A_{\mu\mu}$ is a coefficient of $\cos \delta$ in eq.(1) and it can be also defined as the difference between the maximum and minimum of $P_{\mu\mu}$ s within all range of δ .

$$A_{\mu\mu} \simeq \frac{(P_{\mu\mu}|_{\delta=0^\circ} - P_{\mu\mu}|_{\delta=180^\circ})}{2}. \quad (5)$$

This corresponds to the numerator of I_{CP} [13]. The region with large $A_{\mu\mu}$ will be useful to extract the CP phase. The dependence of $I_{\Delta m_{31}^2}$ and $A_{\mu\mu}$ on L at $E = 1\text{GeV}$ are plotted in Fig.3.

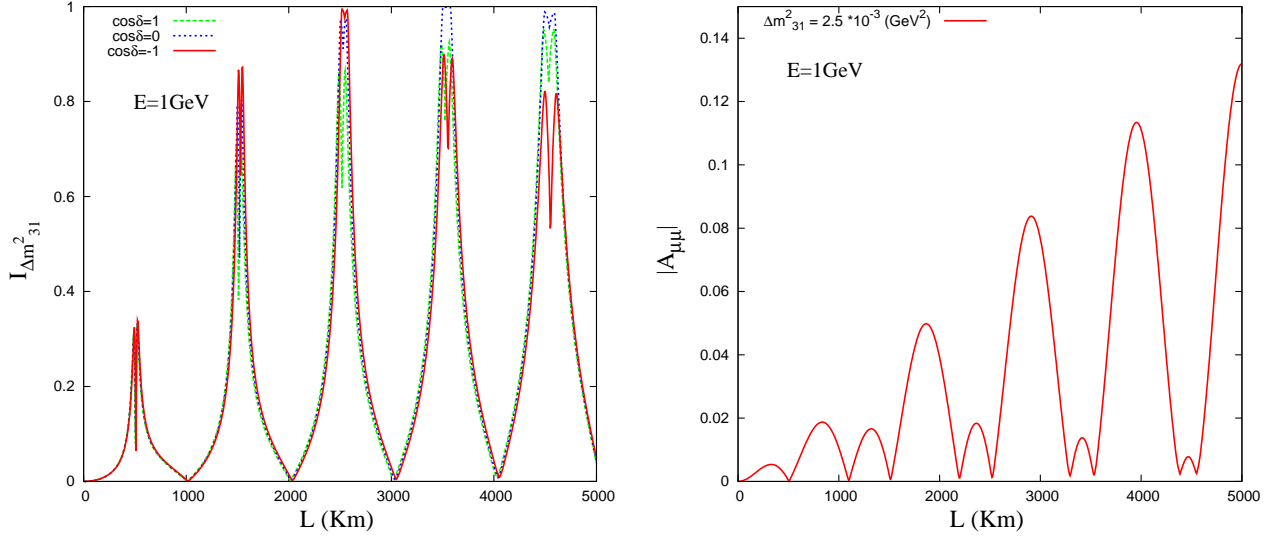


Figure 3: $I_{\Delta m_{31}^2}$ for CP phase $\delta = 0^\circ, 90^\circ, 180^\circ$ (left) and $A_{\mu\mu}$ (right) as the function of baseline length L at $E = 1.0\text{GeV}$.

From Fig.3, around 1000, 2000, 3000, 4000, 5000, ...km, $I_{\Delta m_{31}^2}$ become minimum and then $A_{\mu\mu}$ are showing nonzero values and not so small value. It means that around them, it may be possible to detect the CP phase without depending on Δm_{31}^2 so strong.

The same discussion on the (E, L) plane leads to the better setup to extract CP angle. In Fig. 4, the region are shown as yellow(red) area shows $A_{\mu\mu} > 0.01$ (0.1) and gray is $I_{\Delta m_{31}^2} < 0.05$. From Fig.4, one can roughly estimate the better experimental setup to detect CP phase without depending on the error of Δm_{31}^2 so strong. Around 1000km which means T2KK[4], around 0.5GeV or 1GeV is better energy region. Indeed, longer L is favored for $A_{\mu\mu}$ but the minimal values of $I_{\Delta m_{31}^2}$ will depart from zero so that we must more carefully choose the best place². In addition, we define R_I as the ratio between $I_{\Delta m_{31}^2}$ and $A_{\mu\mu}$ as following,

$$R_I \equiv \frac{I_{\Delta m_{31}^2}}{|A_{\mu\mu}|}. \quad (6)$$

Around the (E, L) where the ratio is smaller than 1 the dependence of $P_{\mu\mu}$ on Δm_{31}^2 should be smaller than the effect by CP phase. In Fig.5, the region of small R_I are plotted. It may show that we can constrain δ by using the setup of (E, L) .

If the experiments are fixed, taking the small and suitable energy bin size, we can avoid the uncertainty from $|\Delta m_{31}^2|$ - $\cos \delta$ correlation. To estimate where is the better setup of (E, L) , the

²We are investigating the T2KK case by using numerical analysis, which will be reported in the other paper[14].

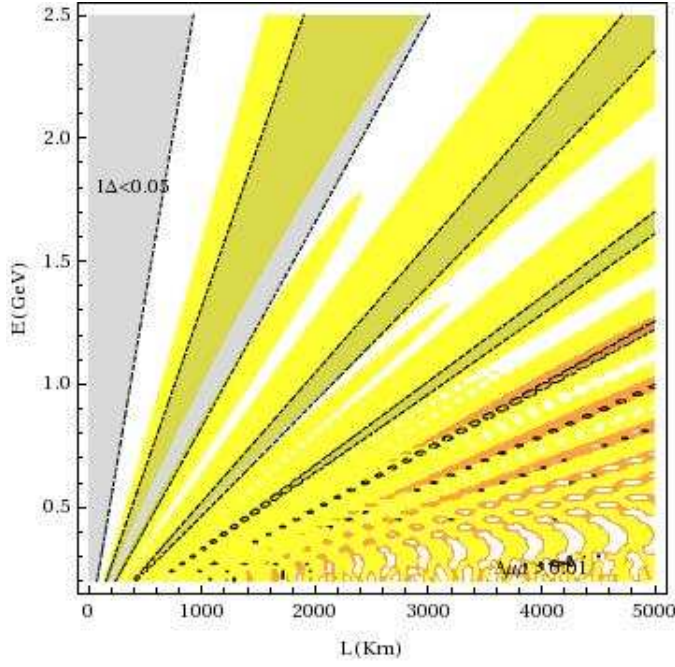


Figure 4: The better region to extract CP effect without depending on the experimental error of Δm_{31}^2 are shown as the overlapping area. The yellow region show $A_{\mu\mu} > 0.01$, the red region is $A_{\mu\mu} > 0.1$ and the gray one is $I_{\Delta m_{31}^2} < 0.05$.

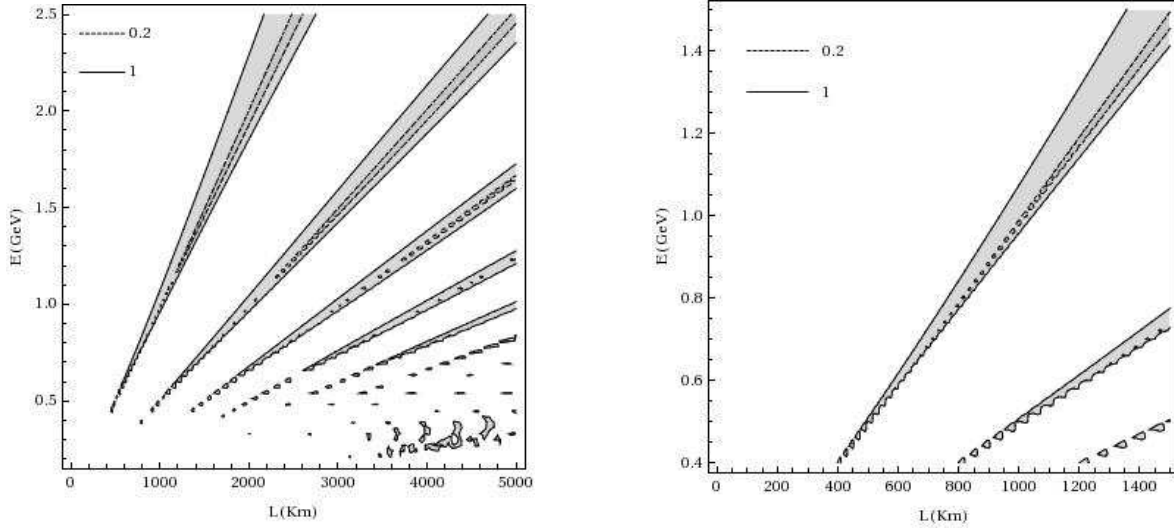


Figure 5: The better region to extract CP effect without depending on the experimental error of Δm_{31}^2 are shown as the area with $R_I < 0.2$ (dashed lines), and 1 (solid line).

new index may be a powerful tool. As you find from Fig.1, even if the error of Δm_{31}^2 is reduced, the uncertainty of δ will remain in almost cases which are not chosen as so good (E, L) . We expect that the new index is going to be such powerful tool to improve the determination of CP phase in $\nu_\mu \rightarrow \nu_\mu$ oscillation and it will be possible to confirm the consistency with the CP effects in $\nu_\mu \rightarrow \nu_e$.

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